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The summation of all prime numbers

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A proof is given of values that can be assigned to the summation of all primes, as well as the summation of all multiples and all odd multiples

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Abstract

A proof is given that the summation of all prime numbers can be assigned the value of $\frac{13}{12}$, as well as values that can be assigned to the summation of all multiples and all odd multiples.

1 Introduction

It is well known that:

$$\sum_{n=0}^{\infty} n = -\frac{1}{12}$$

This result is controversial just as the negative numbers were in the time of Descartes, and the imaginary numbers soon after.
It is clear that we need new definitions to understand this results, we need new concepts to make sense of this truths.
I will do none of that.
Instead I will use this results as we all do now, without much understanding of what they mean but dazzled by they beauty. Specifically I will try to use this result to find the summation over all prime numbers.

2 The proof

We will start from this definition:

$$\sum_{n=0}^{\infty} n = N = 1 + M + P = -\frac{1}{12}$$

Where M stands for the summation of all multiples, P the summation of all prime numbers.
This relationship is true since all numbers, except 1, are either prime or not.
The beginning of M looks like this:

$$M = 4+6+8+9+10+12+14+15+16+18+...+$$

$$M = 2(2+3+4+5+6+7+8+9+...)+9+15+21+...$$

$$M = 2(N-1) + \mu = 2N - 2 + \mu$$

Where now μ stands for the summation of all odd multiples. This is our first restriction which will come up later on.
Now we replace the value of M on the definition of N.

$$N = 1 + 2N - 2 + \mu + P$$

$$N + \mu + P = 1$$

From this previous result we can write our first identity:

$$\mu + P = \frac{13}{12}$$

But we also know that:

$$\mu = M - 2N + 2$$

If we replace this on the first identity we will get the second identity:

$$-N + M + P = -1$$

$$M + P = -\frac{13}{12}$$

Thus we see that the first and second identities tell us of a relationship between P, M and μ, namely:

$$P + \mu = -(P + M)$$

$$2P = -\mu - M$$

Which is the third identity, and our second restriction.
It is impossible to get the value of P from this three identities alone, however they are consistent, and we can try and look for values that satisfies them all, one such group of values that would be very satisfying for us would be:

$$P = \frac{13}{12}, \mu = 0, M = -\frac{26}{12}$$

Which were found by trial and error.
This values work every time, they satisfy every relationship, but they are not unique solutions, for lets consider that some other values were possible, for example:

$$P = \frac{13}{12} + a_1, \mu = a_2, M = -\frac{26}{12} + a_3$$

With this values in the two identities we would find that:

$$a_1 = -a_3 = -a_2$$

And since it is all the same value, lets call it just "a".

It is now our task to fin the value of "a". It would be convenient if it were 0, then it would be true that $\mu = -\mu$, and we can check for that:

First we will do $N - \mu$

$$N - \mu = 1 + 2N - 2 + \mu + P - \mu$$
$$-\frac{1}{12} + a = -1 + 2\Big(-\frac{1}{12}\Big) + \frac{13}{12} + a$$
$$-\frac{1}{12} = -\frac{12}{12} + -\frac{2}{12} + \frac{13}{12}$$
$$-\frac{1}{12} = -\frac{1}{12}$$

Now lets do $N + \mu$

$$N + \mu = 1 + 2N - 2 + \mu + P + \mu$$
$$-\frac{1}{12} - a = -1 + 2\Big(-\frac{1}{12}\Big) + 2\mu + \frac{13}{12} + a$$
$$-\frac{1}{12} - a = -\frac{12}{12} + -\frac{2}{12} + \frac{13}{12} - 2a + a$$
$$-\frac{1}{12} = -\frac{1}{12}$$

From this we find that indeed $\mu = -\mu$, but just to be sure, lets do the same with P:

First $N - P$

$$N - P = 1 + 2N - 2 + \mu + P - P$$
$$-\frac{1}{12} - \frac{13}{12} - a = -1 + 2\Big(-\frac{1}{12}\Big) - a$$
$$-\frac{14}{12} = -\frac{12}{12} + -\frac{2}{12}$$
$$-\frac{14}{12} = -\frac{14}{12}$$

Which is just what we would expect if $P = \frac{13}{12}$, finally $N + P$

$$N + P = 1 + 2N - 2 + \mu + P + P$$
$$-\frac{1}{12} + \frac{13}{12} + a = -1 + 2\Big(-\frac{1}{12}\Big) + 2\Big(\frac{13}{12} + a) - a$$
$$\frac{12}{12} = -\frac{12}{12} + -\frac{2}{12} + \frac{26}{12}$$
$$1 = 1$$

Which, again, is just what we would expect if $P = \frac{13}{12}$
Thus it has been proven that the summation of all prime numbers can be assigned the value of $\frac{13}{12}$, the summation of all multiples is $\frac{-26}{12}$ and the summation of all odd multiples is 0.