Some Techniques for Generating Random variates

2.5 Saila

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Outline

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Here we present useful techniques for generating random variates from a few relatively simple distribution.

Distribution	Parameters	Formula Command in Excel
Bernoulli	p	$X = \begin{cases} 1 & \text{if } U \leq p \\ 0 & \text{if } U > p \end{cases}$
Uniform	$a < b$	$X = a + (b - a)U$ X=a+(b-a)*RAND()
Triangular	$0, \frac{1}{2}, 1$	$X = \frac{1}{2}(U_1 + U_2)$ $X = \frac{1}{2} * (RAND()$ $+RAND())$
Symmetric triangular	$a < b$	$X = a + \frac{(b-a)}{2}(U_1 + U_2)$ $=a + \frac{(b-a)}{2} * (RAND() + RAND())$

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Distribution	Parameters	Formula (in Excel)
Right triangular	$a < c$	$X = a + (c - a)\sqrt{U}$ $X = a + (c - a) * \sqrt{RAND()}$
Approximately normal	0,1	$X = U_1 + U_2$ $+ \cdots + U_{12} - 6$
Approximately normal	$X = \mu, \sigma$	$\mu + \sigma(U_1 + U_2$ $+ \cdots + U_{12} - 6)$
Exponential	μ	$X = -\mu \ln(U)$ $X = -\mu * LN(RAND())$
Discrete uniform	$k, k+1, \ldots, k+m$	$X = k$ $+int[(m+1)U]$

Bernoulli Random variate

The probability density of Bernoulli distribution is given by

$$f(x) = p^x(1-p)^{1-x} \; x = 0, 1 \tag{1}$$

- X has the value 1 with probability p and value 0 with probability $1-p$.

Uniform Random Variates

- Pdf of x, $f(x) = \frac{1}{b-a}$, $b > a$
- A uniformly distributed random variate between ab and b, where $b > a$ can be computed from $X = a + (b-1)U$.
- Use Excel $X = a + (b-a)RAND()$.
- Thus, a uniformly distributed random variate between 0 and 10.0 is given by $X = 10.0u$.
- a uniformly distributed random variate between 20.0 and 100.0 is given by $X = 20.0 + 80.0U$.

Triangular Random Variates

- If U_1 and U_2 are uniformly distributed between 0.0 and 1.0, then $\frac{U_1+U_2}{2}$ has a symmetric triangular distribution between 0 and 1.0.
- If we want a random variate, x to have a symmetric triangular distribution random variate between a and B, X can be computed from

$$X = a + (b-a)(U_1 + U_2) \tag{2}$$

- To generate variates from a nonsymmetric triangular distribution between a and c, where the most likely value is c. X can be computed from

$$X = a + (c-a)\sqrt{U} \tag{3}$$

Evaluating Decision: A one-period Inventory Model

- The ultimate purpose of every model is to predict the likely effects of alternative decision.
- Example: Recall that in chapter 1 (Seila). Suppose we are responsible for deciding how many canister to order for one game. One canister can serve 100 drinks. Let the demand D has an exponential distribution with mean 5.0. The mean of D is expressed in canister, or hundreds of drinks. Let s represent the number of canisters of soft drink that we order. Because D is a random variable, we cannot, for a given value of s, predict whether D will be larger or smaller than s. If D is larger than s, then we will run out, and $D - s$ will be the amount of demand were unable to fill.

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Suppose that each unit (100 drinks) of unmet demand cost us \$40. (We can consider this the cost associated with customer dissatisfaction an unrealized potential profit).Thus the cost associated with ordering s canisters of soft drink, the D is greater than s, is $40.0(D-s)$. Suppose we order too much soft drink so that D is smaller than s. In this case we will have $s-D$ canisters of soft drink left over. Suppose that the cost per unit of excess soft drink is \$10. (This is the cost associated with returning the soft drink to the bottler or otherwise disposing of it). Thus, the cost associated with ordering s canister, when D is less than s is $10.0(s-D)$. We want to determine the value of s that minimizes the expected cost.

Outline

Optimal Order Quantity Using Simulation

To solve this problem

- We first choose a value of s, then set up an experiment in which the value of D is generated from an exponential distribution with mean 5.
- Finally the cost associated with this particular value of s and D is computed.
- This experiment is replicated independently n times, producing n observations of the cost.
- A confidence interval for mean cost is computed from this data.
- The whole process is then repeated for other value of s, producing a confident interval for mean cost for each quantity, s.
- These estimates can be plotted and the plot used to locate the minimum value

Algorithm for simulation

- The algorithm for the simulation is
  1. For $j = 1, 2, \ldots, n$ do:
     To generate D from the distribution of demand
     Compute: $Y_j = \begin{cases} 10.0(s - D), & \text{if } D \leq s \\ 40.0(D - s), & \text{if } D > s \end{cases}$
     Accumulate the sum and sum of squares of the Y_j's.
  2. Compute the sample mean of the Y_j's and a confidence interval for the mean.
- The simulation was run for each values of $s = 5.0, 6.0, \ldots, 12$, using 4000 replications.
- Fig 2.17 gives the results of the simulation and presents them graphically.
- The minimum mean cost appears to be obtained ≈ 8.0 canisters.
- Hillier & Lieberman (1995) solved and determine the exact solution of s, using $F(s*) = \frac{40.0}{40.0+10.0} = 0.80$

Results of multiply runs for inventory model

Problem Parameters:		
Cost per unit for excess		40
Cost per unit for shortage		160
Demand distribution		exponential
Mean demand		5
Decision Variable:		
Order Quantity		8

Simulation:

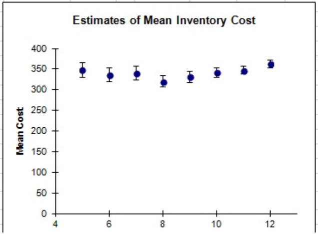
Trial	Demand	Cost
1	1.82	247.01
2	7.17	33.11
3	13.52	882.74
4	0.10	315.95
5	3.26	189.55
6	1.85	245.94
7	1.05	278.04
8	1.72	251.04
9	8.74	117.72
10	1.64	254.49
11	11.23	516.42
12	0.07	317.21
13	5.71	91.55
14	12.55	728.56
15	1.20	272.11
16	14.50	1040.29
17	4.61	135.76
18	3.99	160.48
19	2.18	232.88
20	6.70	52.12
21	5.32	107.19
22	0.42	303.26
23	2.40	224.01
24	2.35	226.05
25	0.26	309.55
26	7.77	9.35

Data Analysis:

Trials	4000
Mean	318.45
Std. Dev.	439.09
Std. Error	6.94
Error	13.61
Lower c.l.	304.84
Upper c.l.	332.06
Min	0.09
Max	5858.86

Decision Analysis

	Mean	Error
Order Qty	318.45	13.61
5	349.512	17.86
6	337.874	17.04
7	341.674	16.15
8	320.564	13.55
9	332.326	13.46
10	343.158	11.62
11	348.008	9.79
12	363.058	9.41



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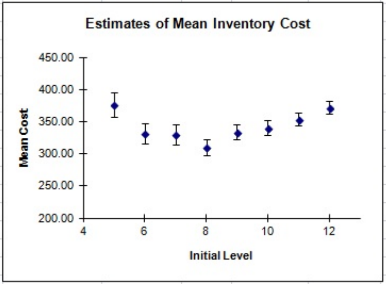
M	N	O	P	Q	R
				95-% Confidence	
	Order Quantity	Mean Cost	Error	Lower Lim	Upper Lim
	5	375.88	19.38	356.50	395.26
	6	331.71	15.93	315.78	347.64
	7	329.75	15.37	314.38	345.12
	8	310.23	12.88	297.35	323.11
	9	334.08	12.19	321.89	346.28
	10	340.55	11.53	329.02	352.08
	11	354.11	10.40	343.71	364.51
	12	372.25	9.73	362.52	381.99



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- Where $F(x)$ is the cumulative distribution function of demand.
- Because the distribution cumulative of demand is exponential, $f(x) = 1 - e^{\frac{x}{5.0}}$, and $s^* = 5\ln(5) = 8.05$.
- We see that the simulation give us a valid and rather accurate to this problem.

Explanation of Statistical Analysis

- Standard error$=\frac{\sigma}{\sqrt{n}}$
- error$=z_{\alpha}\frac{\sigma}{\sqrt{n}}$
- $(1-\alpha)\%$ Confidence interval (C.I) for μ: $\bar{x}-z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\leq\mu\leq\bar{x}+z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
- Lower C.I: $\bar{x}-z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$
- Upper C.I: $\bar{x}+z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}$

If the parent population is normal and σ_x^2 is known.

Standard deviation, σ, using Excel

z_α, using Excel

Solution for Example 2.6.2 (Manual)

1. Construct table of Probabilities and Random number intervals for Daily Ace Drill Demand:

(1) Demand for Ace Drill	(2) Frequency (Days)	(3) Probability	(4) Cumulative Probability	(5) Interval of Rand. Numbers
0	15	0.05	0.05	01 to 05
1	30	0.10	0.15	06 to 15
2	60	0.20	0.35	16 to 35
3	120	0.40	0.75	36 to 75
4	45	0.15	0.90	76 t0 90
5	30	0.10	1.00	91 to 00
	—	—		
	300	1.00		

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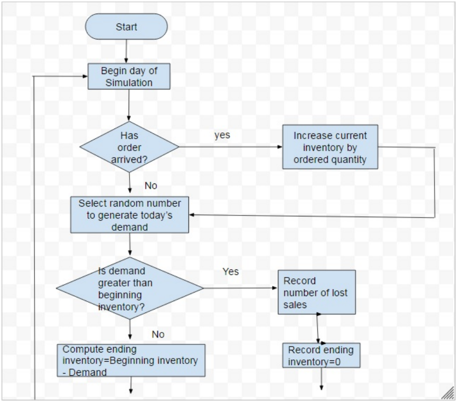
2. Construct table of probabilities and Random Number intervals for Reorder Lead Time:

(1) Lad Time (Days)	(2) Frequency (Orders)	(3) Probability	(4) Cumulative Probability	(5) Random Num. interval
1	10	0.20	0.20	01 to 20
2	25	0.50	0.70	21 to 70
3	15	0.30	1.00	71 to 00
	—	—		
	50	1.00		

The third step

3. Develop the simulation model: A flow diagram, or flowchart, is helpful in the logical coding procedures for programming this simulation process.



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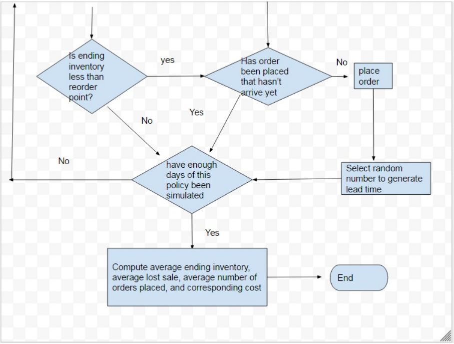

Is ending inventory less than reorder point?
yes
Has order been placed that hasn't arrive yet
No
place order
No
Yes
No
have enough days of this policy been simulated
Select random number to generate lead time
Yes
Compute average ending inventory, average lost sale, average number of orders placed, and corresponding cost
End

Step 4

4. To specify the values that we wish to test.
   - Saad wants to stimulate an order quantity of 10 with a reorder point of 5.
   - Every time the on-hand inventory level at the end of the day is 5 or less, Saad will call the supplier and place and order for 10 more drills.
   - If the lead time is one day, the order will not arrive the next morning but at the beginning

Step 5

5. To conduct the simulation, and the Monte Carlo method is used for this.

- The entire process is simulated for a 10-day period.
- The following table is filled by proceeding on day (or line) at a time, working from left to right. It is a four-step process.
  1. Begin each simulated day by checking whether any ordered inventory has just arrived(column 2). If has increase the current inventory(in column 3) by quantity ordered (10 units, in this case.
  2. Generate a daily demand from the demand probability by selecting a random number. This random number is recorded in column 4. The demand simulated is recorded in column 5.
  3. Compute the ending inventory every day and record it in column 6. Ending inventory=beginning inventory -demand. If on-hand inventory is insufficient to meet the day's demand, satisfy as much as possible and note the number of lost sales (in column 7).
  4. Determine whether the day's ending inventory

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has reached the reorder point (5 units). If it has and if there are no outstanding orders , place an order (column 8). Lead time for a new order is simulated by choosing a random variable from the given table and recording in column 9. We can continue down the same string of random number table that we were using to generate numbers for the demand variable.. Finally, we convert this random variable into a lead time by using the distribution set in the given table.

Simulation

	A	B	C	D	E	F	G	H	I	J	K
1	Saad Electric										
2											
3		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
4		Day	Units Received	Beginning Inventory	Random Number	Demand	Ending Inventory	Lost Sales	Order	Random Number	Lead Time
5		1	----	10	11	1	9	0	No		
6		2	0	9	89	4	5	0	Yes	29	2
7		3	0	5	87	4	1	0	Yes	74	3
8		4	0	1	59	3	0	2	Yes	80	3
9		5	10	10	66	3	7	0	No		
10		6	0	7	53	3	4	0	Yes	73	3
11		7	10	14	45	3	11	0	No		
12		8	10	24	56	3	21	0	No		
13		9	0	21	22	2	19	0	No		
14		10	0	21	49	3	18	0	No		
15					Average=	2.9	9.5	0.2	No		

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a) The average daily demand$=\frac{1+4+4+3+3+3+3+3+2+3}{10}=\frac{29}{10}=2.9\approx 3$ units per day

b) The average lost sales$=\frac{2\ sales\ lost}{10}=0.2$ unit per day.

c) The number of order placed=3 orders

d) The probability that demand per day that exceed 3 units$=\frac{2}{10}=0.2$