Analysis of Material Properties under Bending Load

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Abstract

In this experiment, we attempt to better understand how materials properties are tested. We tested a number of simple beams of different materials under a stress. The bending of the materials allowed for us to calculate the Poisson's Ratio and elastic moduli for each material. From this, we were able to not only compare materials but also methods of measuring elasticity. Despite some error in our results, which can be explained by the scale of our measurements in relation to the stiffness of certain materials, we find both strain gauges and equations of cantilever to be appropriate measurement techniques for measuring the elastic modulus of simple beams.

I. INTRODUCTION

When any material is subjected to a force, that force is distributed throughout the total area of the object. We refer to this distribution of force on the material as stress, and is defined in Equation 1. Stress, as a function of force and area, will be measured in pounds per inches squared, or psi.

$$\sigma = \frac{F}{A} \tag{1}$$

When stress is applied on an object, the object must react accordingly. Thus, when force is applied to a beam, the beam bends. This bending, or any reaction to a force across a material, is called strain[1]. The nominal tensile strain is defined as the extension of a material compared to its length. We represent this concept as ϵ. Stressed materials do not, however, only strain in the lateral direction. Any lateral strain is accompanied by tensile strain. The ratio between the two is called the Poisson's ratio, and is expressed as ν in Equation 2.

$$\nu = \frac{\epsilon_{lateral}}{\epsilon_{tensile}} \tag{2}$$

Now that we know how a material will react to stress compared to itself, it is possible for us to define how different materials will act to the same stress. We call this the elastic modulus, and is defined as the stress on an object versus the strain it experiences. This varies for every material, allowing us to tailor our material choices to the needs of the application. The elastic modulus, in terms of stress and strain, is shown in Equation 3.

$$E = \frac{\sigma}{\epsilon} \tag{3}$$

In this experiment, we will attempt to determine the elastic modulus three ways. In the first method, the stress and strain will be determined, plotted, and the slope of the plot will give us what is called the Young's Modulus. The final two methods stem from Equation 3. Using data taken from our materials, we will use Equation 4 to solve for the stress[2]. In this equation, F is force applied, L_i is the length from the strain measuring guide to the applied load, and b and h represent the base and height of the beam measured.

$$\sigma = \frac{6FL_i}{bh^2} \tag{4}$$

Sample	Height (in)	Base (in)	Total Length (in)	Strain Gauge to Load (in)
GFRP	0.2528	1.04	12	11.47
Wood	0.5225	0.983	12.3	11.8
Aluminum	0.5	1.09	12.25	11.75
Steel	0.184	1.1	12.23	11.73

Table I. Dimensional data for four materials tested

Alternatively, we will use Equation 5 which allows us to measure stress from the displacement of the material being stressed. This gives us our third method of calculating what we will call the average modulus. L represents the length from the support to the load, while δL represents the displacement of the beam due to the load.

$$E = \frac{4FL^3}{\delta L b h^3} \tag{5}$$

II. MATERIALS AND PROCEDURE

In this experiment, four materials were tested by the MMAE 372 Friday Lab group. Glass fiber reinforced polymer (GFRP), wood, steel and aluminum were all put under similar strain conditions meant to stress them across the spectrum of their elastic load limit in order to collect the necessary stress, strain, and displacement data. The height, base, total length, and length from strain gauge to load was measured for each beam, and is displayed in Table 1.

In order to measure stress, strain, and displacement, the material is fixed to a support wall and subjected to a skyward load. Strain gauges were placed along the top and bottom to measure transverse and longitudinal strain. A diagram of the set up is pictured in Figure 1. Figure 2 shows a picture of the experimental set-up, with the displacement measurement device visible. Three pieces of data could be taken under a given load in this set up: the load in pounds-force, the displacement of the beam in inches, and the four-axis strain.

Each sample was tested at fifteen different loads across its elastic spectrum, in order to give us the widest data set possible. From this data, calculations were made in accordance with the before mentioned equations, providing information on the Poisson's Ratio and elastic moduli.

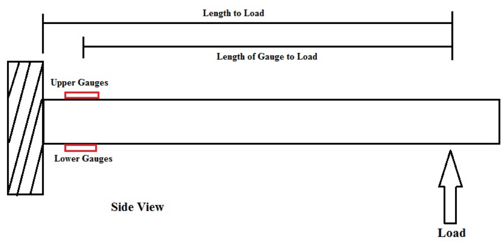

Figure 1. Diagram of the system

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Figure 2. Picture of the system, with displacement register. Wood sample

III. RESULTS

To determine the results of the experiment, Poisson's Ratio was first calculated by plotting longitudinal and transverse strain. A fit of the data was taken, and from this fit, a Poisson's ratio could be determined. The plots and fit equations, of which the slope is the Poisson's Ratio, is pictured in Figure 3.

Figure 4 represents the results of determining the Young's Modulus. The difference between the Young's Modulus and other elastic modlui is that Young's is the slope of a stress strain curve. Thus, longitudinal strain was plotted against stress, measured both on

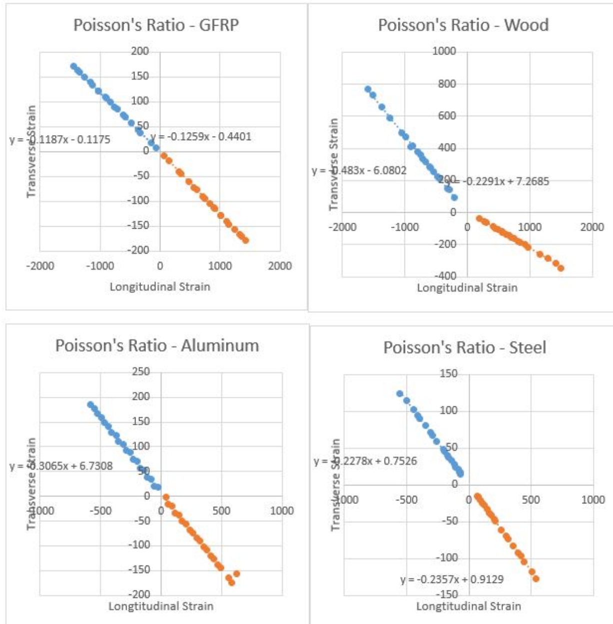

Figure 3. Poisson's Ratio, top (blue) and bottom (orange) strain considered, for four materials. The slope indicated by the fit equations represents Poisson's Ratio.

the top and bottom of the materials, and again, a fit was established from this data.

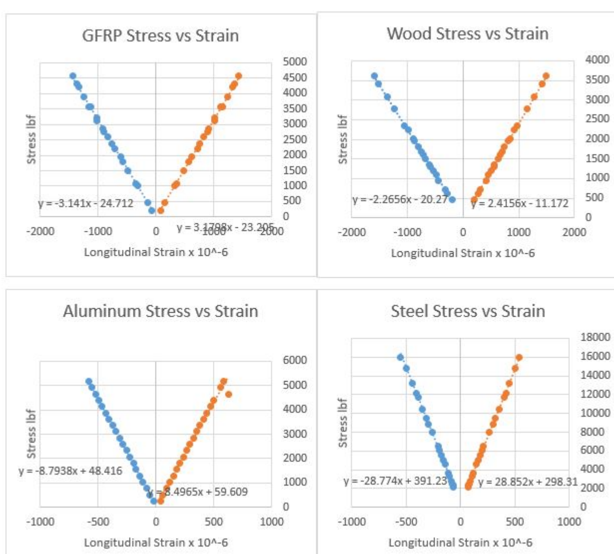

Figure 4. Stress versus Strain for all four materials, the Young's Modulus being the slope of the fit equation of each line.

Our final two methods of calculation came in using Equation 3, 4, and 5 to determine the average elastic modulus by strain (Equation 4 into Equation 5) and by displacement

Sample	Poisson's Ratio - Top	Poisson's Ratio - Bottom	E - Top (psi X 10^6)	E - Bottom (psi X 10^6)	Average E by Strain (psi X 10^6)	Average E by Displacement (psi X 10^6)
GFRP	-0.1187	-0.1259	3.141	3.1798	3.08	2.80
Wood	-0.483	-0.229	2.2656	2.4156	2.30	1.72
Aluminum	-0.3065	-0.2934	8.7938	8.4965	8.97	8.02
Steel	-0.2287	-0.2357	28.774	28.852	30.50	69.51

Table II. Poisson's Ratio and Elastic Modulus Results for all materials

(Equation 6). These results, based not on fit but on calculation of the data given by equations and then averaged, is summed with all our findings in Table II.

IV. DISCUSSION

From the determined results of the experiment, three conclusions can be made: First, Poisson's Ratio, or the rate of strain-longitudinal to strain-traversal is rather constant. We find that the solutions are nearly identical in three cases (GFRP: factor of .94, Aluminum: .96, Steel: .97), with the outlier being wood at a factor of .47 between top and bottom. The plot of Poisson's Ratio makes this clear, and we attribute this to the possibility that wood, being the most porous and least dense of all materials surveyed, may posses unique properties for distributing strain across itself. Perhaps the strain "dissipates" into the porous space of the wood. Its natural flexibility allows it to bend less uniformly. For the metal and poly carbonates, however, we find these more brittle elements to strain quite uniformly.

Second, we see that the four different materials posses a wide range of elastic modulus values, as to be expected. What we traditionally think of as the "stronger" materials (aluminum, steel), exhibited greater resistance to stress. Metals, particularly steel, which has likely undergone a more rigorous hardening process than aluminum, possessed a modulus nearly 10 times as great. This is why steel is so widely used in rigid construction, while aluminum is more likely to be used in mass-production capacities where resistance to stress is not an issue.

Finally, we observe an interesting discrepancy in the relationship between the Young's Modulus (E) and the two methods for calculating average elastic modulus. If we consider Young's Modulus as the most accurate measure as assume two things: the linear fit of the data takes the stronger average and that the information regarding max elastic load was correct and not exceeded. If this is correct, we must judge strain-based average elastic modulus versus displacement-based average elastic modulus versus the fit of the Young's Modulus. We find that in almost all cases, displacement-based calculations were less accurate, generally returning too low of a value, or in the case of steel, too high.Sources of error for the displacement method may be that as the beams bend more and more, as steel did, the one-axis displacement gauge is incapable of measuring the true change in displacement. This, in conjunction with the knowledge that the strain gauge was capable of measuring at a level of specificity far beyond the displacement gauge, allows us to conclude that strain data and Equation 4 are a superior means of calculating the average elastic modulus.

V. CONCLUSIONS

In conclusion, this experiment was intended to provide familiarization with stress, strain, and displacement measuring methods and equipment, by calculating and making observations of the Poisson's Ratio and the different methods of determining the elastic modulus of different materials. The materials in question all performed rather consistently, as shown by Figures 3 and 4. The results, as shown in Table II, are generally in accordance with our perceptions of the materials tested, with exceptions likely due to measurement equipment limits or the natural properties of the samples tested. We ultimately find that Poisson's Ratio generally remains constant across a beam of the tested size from top to bottom, and when using the Young's Modulus as a benchmark for the elastic modulus, strain-based calculations provide the next most desirable average elastic modulus. Further experiment should be done to further test the displacement properties of steel, and theories regarding the porous nature of wood.

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[1] "Engineering Materials I" Fourth Edition, Elsevier Publishing, Michael Ashby and David Jones, pages 30-36, Accessed 29 January 2016

[2] MMAE 372 Lab 1 Instructions, IIT, Dr. Sammy Tin, Accessed 22 January 2016